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Electromagnetic baryon form factors from holographic QCD

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Abstract: In the holographic model of QCD suggested by Sakai and Sugimoto, baryons are chiral solitons sourced by D4 instantons in bulk of size $1/\sqrt{\lambda}$ with $\lambda = g^2 N_c$. We quantize the D4 instanton semiclassically using $\hbar = 1/(N_c\lambda)$ and non-rigid constraints on the vector mesons. The holographic baryon is a small chiral bag in the holographic direction with a Cheshire cat smile. The vector-baryon interactions occur at the core boundary of the instanton in $D4$. They are strong and of order $1/\sqrt{\hbar}$. To order $\hbar^0$ the electromagnetic current is entirely encoded on the core boundary and vector-meson dominated. To this order, the electromagnetic charge radius is of order λ^0. The meson contribution to the baryon magnetic moments sums identically to the core contribution. The proton and neutron magnetic moment are tied by a model independent relation similar to the one observed in the Skyrme model.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence.

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Contents

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1. Introduction

Holographic QCD has provided an insightful look to a number of issues in baryonic physics at strong coupling $\lambda = g^2 N_c$ and large number of colors N_c [1−7]. In particular, in [1, 2] baryons are constructed from a five-dimensional Shrodinger-like equation whereby the 5th dimension generates mass-like anomalous dimensions through pertinent boundary conditions. A number of baryonic properties have followed ranging from baryonic spectra to form factors [1, 2].

At large N_c baryons are chiral solitons in QCD. A particularly interesting framework for discussing this scenario is the D8-$\overline{\rm D8}$ chiral holographic model recently suggested by Sakai and Sugimoto [3, 8, 9] (herethrough hQCD). In hQCD D4 static instantons in bulk source the chiral solitons or Skyrmions on the boundary. The instantons have a size of order $1/\sqrt{\lambda}$ and a mass of order $N_c\lambda$ in units of $M_{\rm KK}$, the Kaluza-Klein scale [3]. The static Skyrmion is just the instanton holonomy in the Z-direction, with a larger size of order λ^0 [10].

In this paper we would like to elaborate further on the precedent observation by explicitly constructing the pertinent electromagnetic current for a holographic soliton following from the exact D4 instanton in bulk in a semiclassical expansion with $\hbar = 1/\lambda N_c$. The vector mesons are quantized using non-rigid constraints to preserve causality. The electromagnetic current is boundary valued as expected from the solitonic nature of the baryon as well as the holographic principle. To order $\hbar^0$ the current is entirely vector meson dominated in overall agreement with the effective analysis in [2]. Our semiclassical analysis provides a book-keeping framework for analyzing the baryons in holographic QCD. It also clarifies a recent analysis [4].

We note that the instanton dynamics in bulk follows from the reduced DBI action. For instantons of size $1/\sqrt{\lambda}$, their field strengths are large and of order λ. In a way the use of the reduced DBI action is not justified. However, since the analysis to follow relies on semiclassics, we believe that our final results are generic enough to hold for the exact instanton solution as well.

In section 2 we briefly go over the soliton-instanton configuration of the Yang-Mills-Chern-Simons effective theory of the Sakai-Sugimoto model in 5 dimensions, including some generic symmetries of the instanton configuration. In section 3 we detail our semiclassical analysis to order $\hbar^0$. In section 4 we derive the baryon current also to order $\hbar^0$, and show that it is vector meson dominated. In section 5 we derive the electromagnetic form factor and show that the minimum and magnetic vector couplings are tied by the solitonic nature of the baryon to order $\hbar^0$. In section 6 the electromagetic charge and radius are worked out. While the instanton in bulk carries a size of order $1/\sqrt{\lambda}$, its holographic image the baryon carries a size of order λ^0 thanks to the trailing vector mesons. The baryon magnetic moments are given in section 7. Our conclusions are in section 8. Some useful details can be found in the appendices.

2. 5D YM-CS model

2.1 Action

In this section we review the action and its soliton solution obtained in [3]. We start with the Yang-Mills-Chern-Simons(YM-CS) theory in a 5D curved background, which has been derived as an effective theory of Sakai-Sugimoto(SS) model [8, 3]. The 5D Yang-Mills action is the leading terms in the $1/\lambda$ expansion of the DBI action of the D8 branes after integrating out the S^4. The 5D Chern-Simons action is obtained from the Chern-Simons action of the D8 branes by integrating F_4 RR flux over the S^4, which is nothing but N_C. The action reads [8, 3]

$$S = S_{\rm YM} + S_{\rm CS}\,, \tag{2.1}$$

$$S_{\rm YM} = -\kappa \int d^4x dZ \;\mathrm{tr}\left[\frac{1}{2}K^{-1/3}\mathcal{F}_{\mu\nu}^2 + M_{\rm KK}^2 K \mathcal{F}_{\mu Z}^2\right]\,, \tag{2.2}$$

$$S_{\rm CS} = \frac{N_c}{24\pi^2}\int_{M^4\times R}\omega_5^{\mathrm{U}(N_f)}(\mathcal{A})\,, \tag{2.3}$$

where $\mu, \nu = 0, 1, 2, 3$ are 4D indices and the fifth(internal) coordinate Z is dimensionless. There are three things which are inherited by the holographic dual gravity theory: $M_{\rm KK}, \kappa$, and K. $M_{\rm KK}$ is the Kaluza-Klein scale and we will set $M_{\rm KK} = 1$ as our unit. κ and K are defined as

$$\kappa = \lambda N_c \frac{1}{216\pi^3} \equiv \lambda N_c a\,, \qquad K = 1 + Z^2\,. \tag{2.4}$$

$\mathcal{A}$ is the 5D $\mathrm{U}(N_f)$ 1-form gauge field and $\mathcal{F}_{\mu\nu}$ and $\mathcal{F}_{\mu Z}$ are the components of the 2-form field strength $\mathcal{F} = \mathrm{d}\mathcal{A} - i\mathcal{A} \wedge \mathcal{A}$. $\omega_5^{\mathrm{U}(N_f)}(\mathcal{A})$ is the Chern-Simons 5-form for the $\mathrm{U}(N_f)$ gauge field:

$$\omega_5^{\mathrm{U}(N_f)}(\mathcal{A}) = \mathrm{tr}\left(\mathcal{A}\mathcal{F}^2 + \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right), \tag{2.5}$$

Since $\mathcal{A}$ is $\mathrm{U}(N_f)$ valued, it may be decomposed into an $\mathrm{SU}(N_f)$ part(A) and a $\mathrm{U}(1)$ part($\widehat{A}$),

$$\mathcal{A} = A + \frac{1}{\sqrt{2N_f}}\widehat{A}\,, \qquad \mathcal{F} = F + \frac{1}{\sqrt{2N_f}}\widehat{F}\,, \tag{2.6}$$

where $A \equiv A^a T^a, F \equiv F^a T^a$ and the $\mathrm{SU}(N_f)$ generators T^a are normalized as

$$\mathrm{tr}\,(T^a T^b) = \frac{1}{2}\delta^{ab}\,. \tag{2.7}$$

For $N_f = 2$ the action (2.2) and (2.3) are reduced to

$$\begin{aligned} S_{\rm YM} &= -\kappa \int d^4x dZ\ \mathrm{tr}\left[\frac{1}{2}K^{-1/3}F_{\mu\nu}^2 + KF_{\mu Z}^2\right] \\ &\quad -\frac{\kappa}{2}\int d^4x dZ\ \left[\frac{1}{2}K^{-1/3}\widehat{F}_{\mu\nu}^2 + K\widehat{F}_{\mu Z}^2\right], \end{aligned} \tag{2.8}$$

$$S_{\rm CS} = \frac{N_c}{24\pi^2}\int\left[\frac{3}{2}\widehat{A}\mathrm{tr}\,F^2 + \frac{1}{4}\widehat{A}\widehat{F}^2 + \frac{1}{2}\,\mathrm{d}\left\{\widehat{A}\ \mathrm{tr}\left(2FA + \frac{i}{2}A^3\right)\right\}\right] \tag{2.9}$$

$$\begin{aligned} &= \frac{N_c}{24\pi^2}\epsilon_{MNPQ}\int d^4x dZ\left[\frac{3}{8}\widehat{A}_0\mathrm{tr}\,(F_{MN}F_{PQ}) - \frac{3}{2}\widehat{A}_M\mathrm{tr}\,(\partial_0 A_N F_{PQ})\right. \\ &\quad +\frac{3}{4}\widehat{F}_M N\mathrm{tr}\,(A_0 F_{PQ}) + \frac{1}{16}\widehat{A}_0\widehat{F}_{MN}\widehat{F}_{PQ} - \frac{1}{4}\widehat{A}_M\widehat{F}_{0N}\widehat{F}_{PQ} \\ &\quad \left.+\frac{3}{2}\partial_N(\widehat{A}_M\mathrm{tr}\,A_0F_{PQ})\right] + \frac{N_c}{48\pi^2}\int\mathrm{d}\left\{\widehat{A}\ \mathrm{tr}\left(2FA + \frac{i}{2}A^3\right)\right\}, \end{aligned} \tag{2.10}$$

where the SU(2) and $\mathrm{U}(1)$ parts are completely desentangled in the Yang-Mills action. The $\omega_5^{\mathrm{SU}(2)}(A)$ vanishes in the CS action. The action (2.8) and (2.9) yield the 10 coupled equations of motion, of which the D4 instanton is a solution with topological charge 1. The coupled equations are specifically given by

$$\delta A_0 \to\ \kappa\left\{D^\mu\left(K^{-1/3}F_{\mu 0}\right) + D^Z\left(KF_{Z0}\right)\right\} - \frac{N_c}{64\pi^2}\epsilon_{MNPQ}(\widehat{F}_{MN}F_{PQ}) = 0\,, \tag{2.11}$$

$$\delta A_i \to\ \kappa\left\{D^\mu\left(K^{-1/3}F_{\mu i}\right) + D^Z\left(KF_{Zi}\right)\right\} - \frac{N_c}{64\pi^2}\epsilon_{iNPQ}(\widehat{F}_{N0}F_{PQ} + \widehat{F}_{PQ}F_{N0}) = 0\,, \tag{2.12}$$

$$\delta A_Z \rightarrow \ \kappa \left\{ D^\mu \left(K F_{\mu Z} \right) \right\} - \frac{N_c}{64\pi^2} \epsilon_{ZNPQ} (\widehat{F}_{N0} F_{PQ} + \widehat{F}_{PQ} F_{N0}) = 0 \,, \tag{2.13}$$

$$\delta \widehat{A}_0 \rightarrow \ \kappa \left\{ \partial^\mu \left(K^{-1/3} \widehat{F}_{\mu 0} \right) + \partial^Z \left(K \widehat{F}_{Z0} \right) \right\} - \frac{N_c}{64\pi^2} \epsilon_{MNPQ} \left(\mathrm{tr} \left(F_{MN} F_{PQ} \right) + \frac{1}{2} \widehat{F}_{MN} \widehat{F}_{PQ} \right) = 0 \,, \tag{2.14}$$

$$\delta \widehat{A}_i \rightarrow \ \kappa \left\{ \partial^\mu \left(K^{-1/3} \widehat{F}_{\mu i} \right) + \partial^Z \left(K \widehat{F}_{Zi} \right) \right\} - \frac{N_c}{16\pi^2} \epsilon_{iNPQ} \left(\mathrm{tr} \left(F_{N0} F_{PQ} \right) + \frac{1}{2} \widehat{F}_{N0} \widehat{F}_{PQ} \right) = 0 \,, \tag{2.15}$$

$$\delta \widehat{A}_Z \rightarrow \ \kappa \left\{ \partial^\mu \left(K \widehat{F}_{\mu Z} \right) \right\} - \frac{N_c}{16\pi^2} \epsilon_{ZNPQ} \left(\mathrm{tr} \left(F_{N0} F_{PQ} \right) + \frac{1}{2} \widehat{F}_{N0} \widehat{F}_{PQ} \right) = 0 \ . \tag{2.16}$$

We note that δA_0 and $\delta \widehat{A}_0$ are constraint type equations or Gauss laws.

2.2 The instanton solution

The exact static $O(4)$ solution in x^M space in the large λ limit is not known. Some generic properties of this solution can be derived for large λ whatever the curvature. Indeed, since $\kappa \sim \lambda$, the instanton solution with unit topological charge that solves (2.11)–(2.16) follows from the YM part only in leading order. It has zero size at infinite λ. At finite λ the instanton size is of order $1/\sqrt{\lambda}$. The reason is that while the CS contribution of order λ^0 is repulsive and wants the instanton to inflate, the warping in the Z-direction of order λ^0 is attractive and wants the instanton to deflate in the Z-direction [2, 3].

For some insights to the warped instanton configuration at large λ we follow [3] and rescale the coordinates and the U(2) gauge fields $\mathcal{A}$ as

$$x^M = \lambda^{-1/2} \widetilde{x}^M \,, \qquad x^0 = \widetilde{x}^0 \,, \qquad \mathcal{A}_M = \lambda^{1/2} \widetilde{\mathcal{A}}_M \,, \qquad \mathcal{A}_0 = \widetilde{\mathcal{A}}_0 \,,$$
$$\mathcal{F}_{MN} = \lambda \widetilde{\mathcal{F}}_{MN} \,, \qquad \mathcal{F}_{0M} = \lambda^{1/2} \widetilde{\mathcal{F}}_{0M} \,, \tag{2.17}$$

where $M, N = 1, 2, 3, Z$ and $x^Z \equiv Z$. The variables with tilde are of order of λ^0. The equations of motions of order λ are

$$\widetilde{D}^N \widetilde{F}_{MN} = 0 \,, \tag{2.18}$$
$$\widetilde{\partial}^N \widehat{\widetilde{F}}_{MN} = 0 \,, \tag{2.19}$$

which yield $\widehat{\widetilde{\mathbb{A}}}_M = 0$[1] for the U(1) part and the BPST instanton solution for the SU(2) part:

$$\widetilde{\mathbb{A}}_M = \eta_{iMN} \frac{\sigma_i}{2} \frac{2 \widetilde{x}_N}{\widetilde{\xi}^2 + \widetilde{\rho}^2} \,, \qquad \widetilde{\mathbb{F}}_{MN} = \eta_{iMN} \frac{\sigma_i}{2} \frac{-4 \widetilde{\rho}^2}{(\widetilde{\xi}^2 + \widetilde{\rho}^2)^2} \ . \tag{2.20}$$

[1] For clarity we summarize our convention here. Greek indices $\{\mu, \nu\} = 0, 1, 2, 3, 4$, capital latin indices $\{M, N, P, Q\} = 1, 2, 3, 4$, and small latin indices $\{i, j, k\} = 1, 2, 3$. The fifth coordinate Z has index 4. i.e. $4 \equiv Z$. The gauge field and field strength with hat are U(1) valued and without hat they are SU(2) valued. All variables with tilde are of order of λ^0 and without tilde they behave as (2.17). We denote the classical field by the boldface. (e.g $\mathbb{A}$). A and F are understood as form without component indices.

JHEP09(2008)007

The instanton is located at the origin so $\widetilde{\xi} \equiv \sqrt{\widetilde{\vec{x}}^2 + \widetilde{Z}^2}$. η_{iMN} is t'Hooft symbol defined as $\eta_{ijk} \equiv \epsilon_{ijk}$, and $\eta_{iMZ} = \delta_{iM}$. At this order $\widetilde{A}_0$ and $\widehat{\widetilde{A}}_0$ are not determined and there is no restriction on the size of the BPST instanton.

The equations of motion to order λ^0 are

$$\widetilde{D}_M^2 \widetilde{A}_0 = 0\,, \tag{2.21}$$

$$\widetilde{\partial}_M^2 \widehat{\widetilde{A}}_0 - \frac{1}{64\pi^2 a}\epsilon_{MNPQ}\mathrm{tr}\,(\widetilde{F}_{MN}\widetilde{F}_{PQ}) = 0\,, \tag{2.22}$$

$$\frac{2}{3}\widetilde{Z}^2\widetilde{D}_j\widetilde{F}_{ij} + 2\widetilde{Z}^2\widetilde{D}_Z\widetilde{F}_{iZ} + 2\widetilde{D}_0\widetilde{F}_{0i} - \frac{1}{8\pi^2 a}\epsilon_{ijkZ}\widehat{\widetilde{A}}_0(\widetilde{D}_j\widetilde{F}_{kZ}) = 0\,, \tag{2.23}$$

$$-2\widetilde{Z}^2\widetilde{D}_i\widetilde{F}_{iZ} + 2\widetilde{D}_0\widetilde{F}_{0Z} - \frac{1}{8\pi^2 a}\epsilon_{ijkZ}\widehat{\widetilde{A}}_0(\widetilde{D}_k\widetilde{F}_{ij}) = 0\,, \tag{2.24}$$

Gauss law (2.21) and (2.22) fix $\widetilde{\mathbb{A}}_0$ and $\widehat{\widetilde{\mathbb{A}}}_0$ as

$$\widetilde{\mathbb{A}}_0 = 0\,, \qquad \widehat{\widetilde{\mathbb{A}}}_0 = -\frac{1}{8\pi^2 a}\frac{2\widetilde{\rho}^2 + \widetilde{\xi}^2}{(\widetilde{\rho}^2 + \widetilde{\xi}^2)^2}\,. \tag{2.25}$$

To this order, the leading BPST solution together with (2.25) solve (2.23) and (2.24) for fixed size $\tilde{\rho}$ of order λ^0. Equivalently, this size follows from the the minimum of the energy to order $1/\lambda$ [3]. For completeness, we note in this section that in terms of the unrescaled variables the instanton gauge fields are

$$\mathbb{A}_0 = 0\,, \qquad \mathbb{A}_M = \eta_{iMN}\frac{\sigma_i}{2}\frac{2x_N}{\xi^2 + \rho^2}\,, \tag{2.26}$$

$$\widehat{\mathbb{A}}_0 = -\frac{1}{8\pi^2 a\lambda}\frac{2\rho^2 + \xi^2}{(\rho^2 + \xi^2)^2}\,, \qquad \widehat{\mathbb{A}}_M = 0\,, \tag{2.27}$$

and the nonvanishing field strengths are

$$\mathbb{F}_{MN} = \eta_{iMN}\frac{\sigma_i}{2}\frac{-4\rho^2}{(\xi^2 + \rho^2)^2}\,, \qquad \widehat{\mathbb{F}}_{M0} = \frac{x^M(3\rho^2 + \xi^2)}{4\pi^2 a\lambda(\rho^2 + \xi^2)^3}\,, \tag{2.28}$$

with the size $\rho = \tilde{\rho}/\sqrt{\lambda}$,

$$\rho^2 = \frac{1}{8\pi^2 a\lambda}\sqrt{\frac{6}{5}}\,. \tag{2.29}$$

We note that near the origin $\xi \sim 0$, the field strengths are large with $\mathbb{F} \sim 1/\rho^2 \sim \lambda$ and $\widehat{\mathbb{F}} \sim 1/(\lambda\rho^4) \sim \lambda$. In a way the reduced DBI action (2.2) is not justified for such field strengths since higher powers of the field strength contribute. For our semiclassical analysis below this does not really matter, since an exact solution of the instanton problem with the full DBI action will not affect the generic nature of most results below as we noted in the introduction.

For large Z and finite λ, the warped instanton configuration is not known. While we do not need it for the semiclassical analysis we will detail below, some generic properties can be inferred. Indeed, the small-Z BPST configuration above has maximal spherical symmetry. That is that an isospin rotation is equivalent to (minus) a space rotation, a feature that is immediately checked through

$$(\mathbb{R}\mathbb{A})_Z = \mathbb{A}_Z(\mathbb{R}\vec{x}) , \qquad (\mathbb{R}^{ab}\mathbb{A}^b)_i = \mathbb{R}^T_{ij}\mathbb{A}^a_j(\mathbb{R}\vec{x}) \ , \tag{2.30}$$

with $\mathbb{R}$ a rigid SO(3) rotation. When semiclassically quantized, the instanton-baryon configuration yields a tower of states with isospin matching minus the spin. This is expected, since the holographic instanton is a Skyrmion on the boundary with hedgehog symmetry. These symmetries can be used to construct variationally the warped instanton configuration, a point we will present elsewhere.

3. Non-rigid semiclassical expansion

In this section we assume that the instanton configuration $\mathbb{A}$ solves exactly the equations of motion for all Z and all λ and proceed to quantize it semiclassically using $\hbar = 1/\kappa \sim 1/\lambda N_c$. For the book-keeping to work we count ρ^2 of order $\hbar^0$. Since the holographic pion decay constant $f_\pi^2 \sim \kappa$, this is effectively the analogue of the semiclassical $1/N_c$ expansion of the boundary Skyrmion, albeit at strong λ coupling.

We now note that $\mathbb{A}$ exhibits exact flavor, translational and rotational zero modes as well as soft or quasi-zero modes in the size ρ and conformal direction Z. We will use collective coordinates to quantize them in general. While for the electromagnetic analysis below we focus on the isorotations (minus the spatial rotations) only, we will discuss in this section the semiclassical anlysis in general.

Generically, we have in the body fixed frame

$$A_M(t, x, Z) = \mathbb{R}(t)\left(\mathbb{A}_M(x - X_0(t), Z - Z_0(t)) + C_M(t, x - X_0(t), Z - Z_0(t))\right) \ , \tag{3.1}$$

with $\rho = \rho(t)$. The classical part transforms inhomogeneously under flavor gauge transformation, while the quantum part transforms homogeneously. The fluctuations C are quantum and of order $\sqrt{\hbar}$ (see below). The isoration $\mathbb{R}$ is an SO(3) matrix which is the adjoint representation of the SU(2) flavor group. Its generators are real $(G^B)^{ab} = \epsilon^{aBb}$. To order $\hbar^0$ the constrained field $\widehat{\mathbb{A}}_0$ remains unchanged, while the constrained field $\mathbb{A}_0 = 0$ shifts by a time-dependent zero mode as detailed in appendix A. The collective coordinates $\mathbb{R}, X_0, Z_0, \rho$ and the fluctuations C in (3.1) form a redundant set. Indeed, the true zero modes

$$\delta^B_R \mathbb{A}_M = G^B \mathbb{A}_M \, , \qquad \delta^i \mathbb{A}_M = \nabla^i \mathbb{A}_M \, , \tag{3.2}$$

and the quasi-zero or soft modes are

$$\delta_Z \mathbb{A} = \partial_Z \mathbb{A} \, , \qquad \delta_\rho \mathbb{A}_M = \partial_\rho \mathbb{A}_M \, , \tag{3.3}$$

modulo gauge transformations. All the analysis to follow will be carried to order $\hbar^0$. To avoid double counting, we need to orthogonalize (gauge fix) the vector fluctuations C from (3.2)–(3.3) though pertinent constraints. In the *rigid quantization*, the exact zero

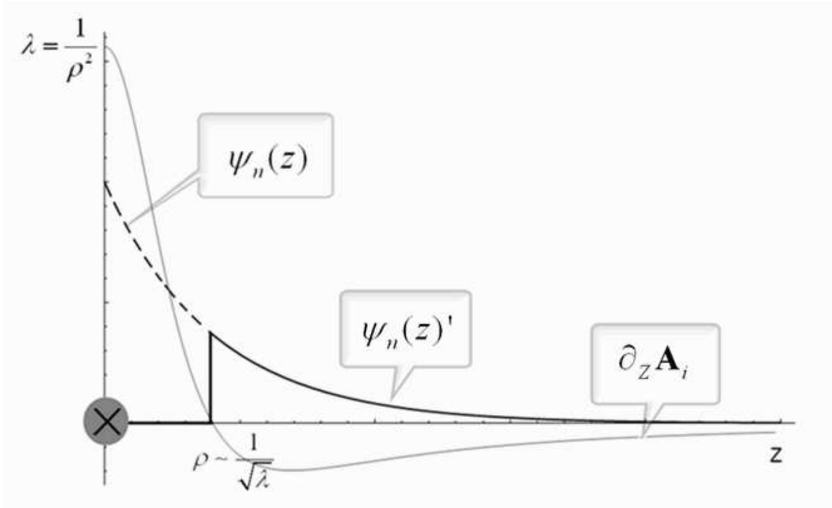

Figure 1: The Z-mode in the non-rigid gauge vs $\partial_Z \mathbb{A}_i$.

modes are removed from the spectrum of C. For example, the isorotations are removed by the constraint

$$\int d\xi\, C\, G^B \mathbb{A}_M \,, \tag{3.4}$$

and similarly for the translations. For an application to chiral baryons of this method we refer to [11]. This constraint violates causality as the fluctuation orthogonalizes instantaneously to an infinitesimal isorotation throughout the instanton body. A causal semiclassical quantization scheme has been discussed in [12]. Here, it means that for instance (3.4) should only be enforced at the location of the instanton, i.e.

$$\int_{x=Z=0} d\hat{\xi} C\, G^B \mathbb{A}_M \ . \tag{3.5}$$

For Z and ρ the non-rigid constraints are more natural to implement since these modes are only soft near the origin at large λ. The vector fluctuations at the origin linearize through the modes

$$d^2\psi_n/dZ^2 = \lambda_n \psi_n \,, \tag{3.6}$$

with $\psi_n(Z) \sim e^{-\sqrt{\lambda_n}Z}$. In the spin-isospin 1 channel they are easily confused with $\partial_Z \mathbb{A}_i$ near the origin as we show in figure 1. Using the non-rigid constraint, the double counting is removed by removing the origin from the vector mode functions

$$\psi_n'(Z) = \theta(|Z| - Z_c)\psi_n(Z) \,, \tag{3.7}$$

with $Z_C \sim \rho \sim 1/\sqrt{\lambda}$ which becomes the origin for large λ. In the non-rigid semiclassical framework, the baryon at small $\xi < |Z_C|$ is described by an instanton located at the origin of R^4 and rattling in the vicinity of Z_C. At large $\xi > |Z_C|$, the rattling instanton sources the vector meson fields described by a semi-classical expansion with non-rigid Dirac constraints. Changes in Z_c (the core boundary) are reabsorbed by a residual gauge transformation on the core instanton. This is a holographic realization of the Cheshire cat principle [13] where Z_c plays the role of the Cheshire cat smile.

For simplicity, and throughout the semiclassical expansion we will ignore the translational zero modes. Also for simplicity, the expansion will not rely on the Dirac constraint of the isorotations since to order $\hbar^0$ their contribution does not arise in the electromagnetic current. A similar observation was made in the Skyrme model where pion-baryon couplings are found to be leading and time-like [14, 12]. At order $\hbar$ and higher the Dirac constraints matter. The constraint on the Z-mode is implemented by Z_C throughout.

To order $\hbar^0$ the semiclassical expansion will be carried out covariantly in the action formalism, whereby Gauss law is unfolded for both $\widehat{\mathbb{A}}_0$ and $\mathbb{A}_0$ as detailed in appendix A. To this order there is no difference between the canonical Hamiltonian formalism, with the advantage of manifest covariance for the derived flavor currents.

Having said this, we now use the gauge field decomposition presented in [9] for the non-rigid semiclassical expansion and refer to this work for further references. Specifically,

$$A_\mu = \mathbb{A}_\mu + C_\mu\,, \qquad C_\mu \equiv v^n_\mu \psi_{2n-1} + a^n_\mu \psi_{2n} + \mathcal{V}_\mu + \mathcal{A}_\mu \psi_0\,, \tag{3.8}$$
$$A_Z = \mathbb{A}_Z + C_Z\,, \qquad C_Z \equiv -i\Pi\phi_0\,, \tag{3.9}$$
$$\widehat{A}_\mu = \widehat{\mathbb{A}}_\mu + \widehat{C}_\mu\,, \qquad \widehat{C}_\mu \equiv \widehat{v}^n_\mu \psi_{2n-1} + \widehat{a}^n_\mu \psi_{2n} + \widehat{\mathcal{V}}_\mu + \widehat{\mathcal{A}}_\mu \psi_0\,, \tag{3.10}$$
$$\widehat{A}_Z = \widehat{\mathbb{A}}_Z + \widehat{C}_Z\,, \qquad \widehat{C}_Z \equiv -i\widehat{\Pi}\phi_0\,, \tag{3.11}$$

where $\{\mathbb{A}, \widehat{\mathbb{A}}\}$ refer to the instanton configuration and $\{C, \widehat{C}\}$ to the vector meson fluctuations. The $\mathbb{R}$ rotation is subsumed. $\{v^n, \widehat{v}^n\}$, $\{a^n, \widehat{a}^n\}$, and $\{\Pi, \widehat{\Pi}\}$ are the vector mesons, the axial vector mesons and the pions respectively. $\{\mathcal{V}, \widehat{\mathcal{V}}\}$ is the vector source and $\{\mathcal{A}, \widehat{\mathcal{A}}\}$ is the axial vector source. Theses meson and source fields are all functions of x^μ. They are attached to the mode functions $\{\psi, \phi\}$ in bulk which are functions of Z as expounded in [9]:

$$-K^{1/3}\partial_Z(K\partial_Z\psi_n) = m^2_{v^n}\psi_n\,, \qquad \kappa\int dZ K^{-1/3}\psi_n\psi_m = \delta_{nm}\,, \tag{3.12}$$
$$\alpha_{v^n} \equiv \kappa\int dZ K^{-1/3}\psi_{2n-1}\,, \qquad \alpha_{a^n} \equiv \kappa\int dZ K^{-1/3}\psi_{2n}\psi_0\,, \tag{3.13}$$
$$\psi_0 \equiv \frac{2}{\pi}\arctan Z\,, \qquad \phi_0 \equiv \frac{1}{\sqrt{\pi\kappa K}}\,. \tag{3.14}$$

With the gauge field (3.8)–(3.11) the SU(2) YM action reads

$$S_{YM} = -\frac{\kappa}{2}\int d^4x dZ\Big[\partial^Z\left(2K\widehat{\mathbb{F}}_{Z\mu}\widehat{C}^\mu\right) + \frac{1}{2}K^{-1/3}\left(\partial_\mu\widehat{C}_\nu - \partial_\nu\widehat{C}_\mu\right)^2 + K\left(\partial_Z\widehat{C}_\mu - \partial_\mu\widehat{C}_Z\right)^2\Big], \tag{3.15}$$

$$-\kappa \int d^4x dZ \mathrm{tr} \Big[\partial^Z \left(2K\mathbb{F}_{Z\nu}C^\nu\right)$$
$$+\frac{1}{2}K^{-1/3}\left\{2\mathbb{F}_{\mu\nu}[C^\mu, C^\nu] + \Big(\mathbb{D}_\mu C_\nu - \mathbb{D}_\nu C_\mu - i[C_\mu, C_\nu]\Big)^2\right\}$$
$$+K\left\{2\mathbb{F}_{Z\mu}[C^Z, C^\mu] + \Big(\mathbb{D}_Z C_\mu - \mathbb{D}_\mu C_Z - i[C_Z, C_\mu]\Big)^2\right\}\Big],$$

where $\mathbb{D}_\alpha$ is the covariant derivative with the slowly rotating instanton in flavor space: $\mathbb{D}_\alpha * = \partial_\alpha - i[\mathbb{A}_\alpha, *]$. We dropped the leading and pure instanton part for convenience. Some details regarding the expansion including the CS part are briefly given in appendix B. All the linear terms to $\{C, \widehat{C}\}$ except the *boundary terms* in the YM and CS action vanish due to the equations of motion. There is no coupling in bulk between the vector mesons and the instanton configuration except through boundary terms and/or time derivatives. This is the hallmark of solitons. While it apparently looks different from the effective and holographic description presented in [2] as couplings are involved in bulk, we will show below that the results are indeed similar for the electromagnetic form factors to order $\hbar^0$. Below, we will explain why the similarity.

We note that without the instanton $\{\mathbb{A}, \widehat{\mathbb{A}}\}$, the action reduces to the one in [9], where the vector meson dominance (VMD) of the pion form factor follows from the field redefinitions

$$v^n \to v^n_{\rm new} = v^n + \alpha_{v^n}\mathcal{V} + \frac{b_{v^n\pi\pi}}{2f_\pi^2}[\Pi, \mathrm{d}\Pi], \tag{3.16}$$

$$b_{v^n\pi\pi} \equiv \kappa \int dZ K^{-1/3}(1-\psi_0^2), \tag{3.17}$$

with f_π the pion decay constant. This redefinition yields a direct vector-photon coupling $v^n - \mathcal{V}$

$$m^2_{v^n}\left(v^n_{\rm new} - \alpha_{v^n}\mathcal{V}\right)^2 , \tag{3.18}$$

while removing all pion-photon couplings $\Pi - \mathcal{V}$ through various sum rules. Holographic QCD obeys the strictures of VMD in the meson sector. This point will carry semiclassically to the baryon sector as we detail below.

Substituting the 5D fields for the 4D fields with mode functions we have to order $\hbar^0$

$$S_{\rm eff} = -\sum_{n=1}^{\infty}\int d^4x \Big[\Big[\kappa K \widehat{\mathbb{F}}^{Z\mu}\Big((\widehat{v}^n_\mu - \alpha_{v^n}\widehat{\mathcal{V}}_\mu)\psi_{2n-1} + (\widehat{a}^n_\mu - \alpha_{a^n}\widehat{\mathcal{A}}_\mu)\psi_{2n} + \widehat{\mathcal{V}}_\mu + \widehat{\mathcal{A}}_\mu\psi_0\Big)\Big]_{Z=B}$$
$$+\frac{1}{4}\Big(\partial_\mu\widehat{v}^n_\nu - \partial_\nu\widehat{v}^n_\mu\Big)^2 + \frac{1}{2}m^2_{v^n}(\widehat{v}^n_\mu - \alpha_{v^n}\widehat{\mathcal{V}}_\mu)^2$$
$$+\frac{1}{4}\Big(\partial_\mu\widehat{a}^n_\nu - \partial_\nu\widehat{a}^n_\mu\Big)^2 + \frac{1}{2}m^2_{a^n}\Big(\widehat{a}^n_\mu - \alpha_{a^n}\widehat{\mathcal{A}}_\mu\Big)^2\Big], \tag{3.19}$$

$$-\sum_{n=1}^{\infty} 2\mathrm{tr}\int d^4x \Big[\big[\kappa K \mathbb{F}^{Z\nu}\left((v^n_\mu - \alpha_{v^n}\mathcal{V}_\mu)\psi_{2n-1} + (a^n_\mu - \alpha_{a^n}\mathcal{A}_\mu)\psi_{2n} + \mathcal{V}_\mu + \mathcal{A}_\mu\psi_0\right)\big]_{Z=B}$$
$$+\frac{1}{4}\Big(\partial_\mu v^n_\nu - \partial_\nu v^n_\mu\Big)^2 + \frac{1}{2}m^2_{v^n}\left(v^n_\mu - \alpha_{v^n}\mathcal{V}_\mu\right)^2$$
$$+\frac{1}{4}\Big(\partial_\mu a^n_\nu - \partial_\nu a^n_\mu\Big)^2 + \frac{1}{2}m^2_{a^n}\left(a^n_\mu - \alpha_{a^n}\mathcal{A}_\mu\right)^2\Big], \tag{3.20}$$

Figure 2: Left: Direct coupling, Right: Vector meson mediated coupling(VMD)

where all meson fields are the redefined fields, $v^n_{\rm new}$ and $a^n_{\rm new}$, but we drop the subscript for simplicity. $[\cdots]_{Z=B}$ will be evaluated at the boundary $Z = B$, which is collectively denoted by $\{\pm\infty, \pm Z_c\}$. We retained only the terms relevant to the baryon form factor. For the complete expansion of the meson fluctuation part we refer to [9].

4. Baryon current

Now, consider the effective action for the $\mathrm{U}(1)_V$ source to order $\hbar^0$

$$S_{\rm eff}[\widehat{\mathcal{V}}_\mu] = \sum_{n=1}^{\infty} \int d^4x \Big[-\frac{1}{4}\Big(\partial_\mu \widehat{v}^n_\nu - \partial_\nu \widehat{v}^n_\mu\Big)^2 - \frac{1}{2} m^2_{v^n} (\widehat{v}^n_\mu)^2 \qquad (4.1)$$
$$-\kappa K \widehat{\mathbb{F}}^{Z\mu} \widehat{\mathcal{V}}_\mu (1 - \alpha_{v^n} \psi_{2n-1})\Big|_{Z=B}$$
$$+ a_{v^n} m^2_{v^n} \widehat{v}^n_\mu \widehat{\mathcal{V}}^\mu - \kappa K \widehat{\mathbb{F}}^{Z\mu} \widehat{v}^n_\mu \psi_{2n-1}\Big|_{Z=B}\Big] ,$$

The first line is the free action of the massive vector meson which gives the meson propagator

$$\Delta^{mn}_{\mu\nu}(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \left[\frac{-g_{\mu\nu} - p_\mu p_\nu / m^2_{v^n}}{p^2 + m^2_{v^n}} \delta^{mn} \right] , \qquad (4.2)$$

in our convention. The rest are the coupling terms between the source and the instanton: the second line is the direct coupling (figure 2(a)) and the last line corresponds to the coupling mediated by the $\mathrm{U}(1)$ (omega, omega', ...) vector meson couplings (figure 2(b)),

$$\kappa K \widehat{\mathbb{F}}^{Z\mu} \widehat{v}^n_\mu \psi_{2n-1} , \qquad (4.3)$$

which is large and of order $1/\sqrt{\hbar}$ since $\psi_{2n-1} \sim \sqrt{\hbar}$. When ρ is set to $1/\sqrt{\lambda}$ after the book-keeping noted above, the coupling scales like $\lambda\sqrt{N_c}$, or $\sqrt{N_c}$ in the large N_c limit taken first.[2]

The direct coupling drops by the sum rule

$$\sum_{n=1}^{\infty} \alpha_{v^n} \psi_{2n-1} = 1 , \qquad (4.4)$$

[2]The reader may object that such strong couplings may upset the semiclassical expansion through perturbative corrections. This is not the case when the Dirac constraints are imposed properly as noted in [12] for the Skyrme model.

following from closure in curved space

$$\delta(Z-Z')=\sum_{n=1}^{\infty}\ \kappa\psi_{2n-1}(Z)\psi_{2n-1}(Z')K^{-1/3}(Z')\ . \tag{4.5}$$

in complete analogy with VMD for the pion [9].

The baryon current is entirely vector dominated to order $\hbar^0$ and reads

$$J_B^{\mu}(x)\ =\ -\sum_{n,m}\ m_{v^n}^2\alpha_{v^n}\psi_{2m-1}\int d^4y\ \kappa K\widehat{\mathbb{F}}_{Z\nu}(y,Z)\Delta_{mn}^{\nu\mu}(y-x)\Big|_{Z=B}\ . \tag{4.6}$$

This point is in agreement with the effective holographic approach described in [2]. The static baryon charge distribution is

$$J_B^0(\vec{x})=-\sum_{n}\ \int d\vec{y}\ \ \frac{2}{N_c}\kappa K\widehat{\mathbb{F}}_{Z0}(\vec{y},Z)\ \Delta_n(\vec{y}-\vec{x})\ a_{v^n}m_{v^n}^2\psi_{2n-1}\ \Big|_{Z=B}, \tag{4.7}$$

with

$$\Delta_n(\vec{y}-\vec{x})\equiv\int\frac{d\vec{p}}{(2\pi)^3}\frac{e^{-i\vec{p}\cdot(\vec{y}-\vec{x})}}{\vec{p}^2+m_{v^n}^2}\,, \tag{4.8}$$

and the extra factor $2/N_c$ comes from the relation between $\widehat{\mathcal{V}}_\mu$ and the baryon number source $\widehat{B}_0(\vec{x})$: by $\widehat{\mathcal{V}}_\mu=\delta_{\mu 0}\frac{\sqrt{2N_f}}{N_c}\widehat{B}_0(\vec{x})$.

5. Electromagnetic current and form factor

In addition to the baryon current discussed above, we now need the flavor or isospin current to construct the electromagnetic current. For that, consider the 4 dimensional effective action for the SU(2)-valued flavor source again to order $\hbar^0$

$$S_{\rm eff}[\mathcal{V}_\mu^a]\ =\ \sum_{b=1}^{3}\sum_{n=1}^{\infty}\int d^4x\Big[-\frac{1}{4}\Big(\partial_\mu v_\nu^{b,n}-\partial_\nu v_\mu^{b,n}\Big)^2-\frac{1}{2}m_{v^n}^2(v_\mu^{b,n})^2 \tag{5.1}$$
$$+\ \alpha_{v^n}m_{v^n}^2 v_\mu^{a,n}\mathcal{V}^{a,\mu}-\kappa K\mathbb{F}^{b,Z\mu}v_\mu^{b,n}\psi_{2n-1}\Big|_{Z=B}\Big]\,,$$

where the direct coupling vanishes due to the sum rule (4.4) and only the VMD part contributes through the SU(2) (rho, rho', . . .) meson couplings

$$\kappa K\mathbb{F}^{b,Z\mu}v_\mu^{b,n}\psi_{2n-1}\,, \tag{5.2}$$

which is large and of order $1/\sqrt{\hbar}$. Again, this coupling is of order $\sqrt{N_c}\lambda^{3/2}$ after the book-keeping. This contribution is similar to (4.1) apart from the SU(2) labels.

The isospin current is,

$$J_{I,a}^{\mu}(x)=-\sum_{n,m}m_{v^n}^2\alpha_{v^n}\psi_{2n-1}\int d^4y\ \ \kappa K\mathbb{F}_{Z\nu}^a(y,Z)\Delta_{mn}^{\nu\mu}(y-x)\Big|_{Z=B}, \tag{5.3}$$

JHEP09(2008)007

From (4.6) and (5.3) the electromagnetic current is given by

$$\begin{aligned} J^{\mu}_{\rm EM}(x) &= J^{\mu}_{I,3}(x) + \frac{1}{2} J^{\mu}_{B}(x) \\ &= -\sum_{n,m} m^2_{v^n} \alpha_{v^n} \psi_{2m-1} \int d^4y \;\; \mathcal{Q}_{\nu}(y,Z) \Delta^{\nu\mu}_{mn}(y-x) \Big|_{Z=B} , \end{aligned} \tag{5.4}$$

with

$$\mathcal{Q}_{\mu}(x,Z) \equiv \kappa K \mathbb{F}^3{}_{Z\mu}(x,Z) + \frac{1}{N_c} \kappa K \widehat{\mathbb{F}}_{Z\mu}(x,Z) \; . \tag{5.5}$$

The electromagnetic charge density is

$$\begin{aligned} J^{0}_{\rm EM}(x) = &-\sum_{n} \int d^4y \; \mathcal{Q}_0(y,Z) \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(y-x)} \frac{m^2_{v^n}}{p^2 + m^2_{v^n}} \alpha_{v^n} \psi_{2n-1} \Big|_{Z=B} \\ &+ \sum_{n} \int d^4y \; \mathcal{Q}_{\mu}(y,Z) \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(y-x)} \frac{p^{\mu}p^{0}}{p^2 + m^2_{v^n}} \alpha_{v^n} \psi_{2n-1} \Big|_{Z=B} , \end{aligned} \tag{5.6}$$

and the static electromagnetic form factor follows readily in the form

$$\begin{aligned} J^{0}_{\rm EM}(\vec{q}) &= \int d\vec{x} e^{i\vec{q}\cdot\vec{x}} J^{0}_{\rm EM}(x) \\ &= -\sum_{n} \int dZ \partial_Z \left[\left(\int d\vec{x} e^{i\vec{q}\cdot\vec{x}} \mathcal{Q}_0(x,Z) \right) \psi_{2n-1} \right] \frac{\alpha_{v^n} m^2_{v^n}}{\vec{q}^{\,2} + m^2_{v^n}} , \end{aligned} \tag{5.7}$$

after setting $p^0 = 0$ in (5.6) so that $\mathcal{Q}_i$ is irrelevant. Recall that the instanton configuration we are using is adiabatically rotating in flavor space. In the charge $\mathcal{Q}_{\prime}$, these rotations generate a velocity dependence to leading order in $\hbar$ which is proportional to the angular momentum upon semiclassical quantization. With this in mind, there is effectively no time-dependence left in the density $\mathcal{Q}_{\mu}(x,Z)$ to leading order in $\hbar$.

We now note that the electromagnetic form factor (5.7) can be rewritten as

$$J^{0}_{\rm EM}(\vec{q}) = \sum_{n} \left(g^{n}_{V,min} + g^{n}_{V,mag} \right) \frac{\alpha_{v^n} m^2_{v^n}}{\vec{q}^{\,2} + m^2_{v^n}} , \tag{5.8}$$

with

$$\begin{aligned} g^{n}_{V,min} &= \int dZ \partial_Z \left[\left(\int d\vec{x} e^{i\vec{q}\cdot\vec{x}} \mathcal{Q}_0(x,Z) \right) \right] \psi_{2n-1} , \\ g^{n}_{V,mag} &= \int dZ \left(\int d\vec{x} e^{i\vec{q}\cdot\vec{x}} \mathcal{Q}_0(x,Z) \right) \partial_Z \psi_{2n-1} , \end{aligned}$$

which are the analogue of the minimal and magnetic coupling used in the effective baryon description of [2]. The solitonic character of the solution implies that the two contributions are *tied* and sum up to a purely surface term in the Z-direction a point that is not enforced in the effective approach [2]. Also our results are organized in $\hbar$ starting from the original D4 instanton.

6. Electromagnetic charge and charge radius

The nucleon electromagnetic form factor is written as a boundary term

$$J^0_{\rm EM}(\vec{q}) = \int d\vec{x} e^{i\vec{q}\cdot\vec{x}} J^0_{\rm EM}(\vec{x})$$
$$= \int d\vec{x} e^{i\vec{q}\cdot\vec{x}} \sum_n \frac{\alpha_{v^n} m_{v^n}^2}{\vec{q}^{\,2} + m_{v^n}^2} \psi_{2n-1}(Z_C) 2\mathcal{Q}_0(\vec{x}, Z_C)\,, \tag{6.1}$$

where the boundary term at $Z = \infty$ vanishes since $\psi_{2n-1} \sim 1/Z$ for large Z. In the limit $q \to 0$ we pick the electromagnetic charge

$$\int d\vec{x}\, e^{i\vec{q}\cdot\vec{x}}\, 2\mathcal{Q}_0(\vec{x}, Z_C)\,, \tag{6.2}$$

due to the sum rule (4.4). Since Z_c will be set to zero ultimatly at large λ, the limits $\lim_{q\to 0}\lim_{Z\to 0}$ will be understood sequentially. To proceed, we need to work out the surface densities $\mathcal{Q}_0$ i.e. the U(1) and SU(2) parts $K\widehat{\mathbb{F}}_{Z0}(\vec{x}, Z_c)$ and $K\mathbb{F}_{Z0}(\vec{x}, Z_c)$ respectively. By the equations of motion (2.14) and (2.11), they read

$$\frac{4}{N_c}\kappa K\widehat{\mathbb{F}}_{Z0}(Z_c) = \int_{-Z_C}^{Z_C} dZ \frac{1}{32\pi^2}\epsilon_{MNPQ}\left({\rm tr}\,(\mathbb{F}_{MN}\mathbb{F}_{PQ}) + \frac{1}{2}\widehat{\mathbb{F}}_{MN}\widehat{\mathbb{F}}_{PQ}\right) + \frac{2}{N_c}\int_{-Z_C}^{Z_C} dZ\kappa K^{-1/3}\partial^i\widehat{\mathbb{F}}_{0i}\,, \tag{6.3}$$

$$2\kappa K\mathbb{F}^a{}_{Z0}(Z_c) = \int_{-Z_C}^{Z_C} dZ\ 2i\kappa\ {\rm tr}\left\{K^{-1/3}\left([\mathbb{F}_{0i}, \mathbb{A}_i] + K[\mathbb{F}_{0Z}, \mathbb{A}_Z]\right)t^a\right\} + \int_{-Z_C}^{Z_C} dZ\kappa K^{-1/3}\partial^i\mathbb{F}^a_{0i} + \int_{-Z_C}^{Z_C} dZ\frac{N_c}{64\pi^2}\epsilon_{MNPQ}(\widehat{\mathbb{F}}_{MN}\mathbb{F}^a_{PQ}). \tag{6.4}$$

The U(1) number density readily integrates to 1 since

$$B = \int d\vec{x} J^0_B(\vec{x}) = \int d\vec{x}\frac{4}{N_c}\kappa K\widehat{\mathbb{F}}_{Z0}(Z_c)$$
$$= \int d\vec{x}\int_{-Z_C}^{Z_C} dZ\frac{1}{32\pi^2}\epsilon_{MNPQ}{\rm tr}\,(\mathbb{F}_{MN}\mathbb{F}_{PQ}) = 1\,, \tag{6.5}$$

as the spatial flux vanishes on R^3_X and the U(1) winding number arezero for a sufficiently localized SU(2) instanton in $R^3_X \times R_Z$. We note that the integrand is manifestly gauge invariant. To contrast, the isovector charge is

$$I^A = \int d\vec{x} 2\kappa K\mathbb{F}^A{}_{Z0}(Z_c) = \int d\vec{x}\int_{-Z_C}^{Z_C} dZ\ 2i\kappa\ {\rm tr}\left\{K^{-1/3}\left([\mathbb{F}_{0i}, \mathbb{A}_i] + K[\mathbb{F}_{0Z}, \mathbb{A}_Z]\right)t^A\right\}\,, \tag{6.6}$$

again after dropping the surface term in R^3_X and the Chern-Simons contribution for a sufficiently localized instanton in $|Z_C|$. Although the integrand in (6.6) is not manifestly gauge invariant, it is only sensitive to a rigid gauge transformation at Z_C which is reabsorbed by gauge rotating the cloud as is explicit from the mode decomposition.

As noted earlier, the D4 instanton has maximal spherical symmetry so that its isospin I^A is just minus its angular momentum J^A,

$$
\begin{aligned}
J^A &= \int d\vec{x} dZ J^{0A} = \int d\vec{x} dZ \epsilon_{Ajk} x^j T^{0k} \\
&= \int d\vec{x} dZ \epsilon_{Ajk} x^j \left[-\kappa K^{-1/3} \mathbb{F}^{l0,a} \mathbb{F}^{lk,a} - \kappa K \mathbb{F}^{Z0,a} \mathbb{F}^{Zk,a} \right] . \qquad (6.7)
\end{aligned}
$$

Both I^A and J^A are driven by the adiabatic rotation $\mathbb{R}$. For the D4 instanton part, it is

$$
\mathbb{A}_R^a = \mathbb{R}^{ab}(t) \mathbb{A}^b , \qquad \dot{\mathbb{A}}_R^a = \left(\dot{\mathbb{R}}(t) \mathbb{R}^{-1}(t) \right)^{ab} \mathbb{A}_R^b , \qquad (6.8)
$$

with $\omega^A G^A \equiv -\dot{\mathbb{R}} \mathbb{R}^{-1}$.[3] The ω's are quantum and of order $\hbar$. Recalling the result for $\mathbb{A}_0^R$ for the constrained field and the zero mode ($\mathbb{Z}^R$) from appendix A, we obtain to leading order

$$
\begin{aligned}
J^A = -I^A &= \int d\vec{x} dZ \epsilon_{Ajk} x^j \left[-\kappa K^{-1/3} \mathbb{F}^{l0,a} \mathbb{F}^{lk,a} - \kappa K \mathbb{F}^{Z0,a} \mathbb{F}^{Zk,a} \right] \qquad (6.9) \\
&= \int d\vec{x} \int_{-Z_c}^{Z_c} dZ \epsilon_{Ajk} x^j \left[-\kappa (\mathbb{D}^M \mathbb{Z}^R)^a \mathbb{F}^{Mk,a} \right] \to -\frac{1}{2} M_0 \rho^2 \, \omega^A \equiv -\mathbb{I} \, \omega^A .
\end{aligned}
$$

The last relation follows from the BPST instanton (2.28). As expected, the core instanton has a moment of inertia $\mathbb{I} = M_0 \rho^2/2$ where $M_0 = 8\pi^2 \kappa$ is the D4 instanton mass in units of M_{KK} and to leading order in $1/\lambda$. $\mathbb{I}$ is of order N_c. Maximum spherical symmetry results in a symmetric inertia tensor.

Finally, the nucleon charge is then

$$
\int d\vec{x} J^0_{\rm EM}(x) = \int d\vec{x} 2\mathcal{Q}_0(x, Z_c) = I_3 + \frac{B}{2} . \qquad (6.10)
$$

While our analysis is to order $\hbar^0$ this normalization should hold to all orders in $\hbar$. The vector meson cloud encodes the exact charges in holography thanks to the exact sum rule (4.4).

The electromagnetic charge radius $\langle r^2 \rangle_{EM}$ can be read from the q^2 terms of the form factor

$$
\langle r^2 \rangle_{\rm EM} = \int d\vec{x} \; r^2 \, 2\mathcal{Q}_0(\vec{x}, Z_c) + 6 \sum_{n=1}^{\infty} \frac{\alpha_{v^n} \psi_{2n-1}(Z_c)}{m_n^2} \int d\vec{x} \; 2\mathcal{Q}_0(\vec{x}, Z_c) , \qquad (6.11)
$$

with $r \equiv \sqrt{(\vec{x})^2}$. The first contribution is from the core, while the second contribution is from the cloud. For a sufficiently localized instanton in bulk the first contribution is of order $1/\lambda$,

$$
\begin{aligned}
\langle r^2 \rangle'_{I=0} &= \int d\vec{x} \; r^2 \, 2\mathcal{Q}_0(\vec{x}, Z_c) \\
&= \frac{3}{2} \rho^2 \frac{Z_c}{\sqrt{Z_c^2 + \rho^2}} \to \frac{3}{2} \rho^2 . \qquad (6.12)
\end{aligned}
$$

[3] We recall that both the BPST instanton and the fluctuations are rotating in the body fixed frame. As noted earlier, the R-labeling of the fields is subsumed.

The meson cloud contribution is of order λ^0. It can be exactly asessed by noting that

$$\begin{aligned}\sum_{n=1}^{\infty}\frac{\alpha_{v^n}\psi_{2n-1}(Z_c)}{m_n^2} &= \int dZ \sum_{n=1}^{\infty}\frac{\psi_{2n-1}(Z_c)\psi_{2n-1}(Z)K^{-1/3}(Z)}{m_n^2} \\ &= \int dZ \langle Z_c|\Box_{\mathbf{C}}^{-1}|Z\rangle\,, \end{aligned} \tag{6.13}$$

where $\Box_{\mathbf{C}}^{-1} \equiv -\partial_Z^{-1}K^{-1}\partial_Z^{-1}K^{-1/3}$ is the inverse of (3.12). This is just the vector meson propagator in curved space.[4] It follows that

$$\sum_{n=1}^{\infty}\frac{\alpha_{v^n}\psi_{2n-1}(Z_c)}{m_n^2} = -\int dZ\int dZ'\langle Z_c|\partial_Z^{-1}|Z'\rangle K^{-1}(Z')\langle Z'|\partial_Z^{-1}|Z\rangle K^{-1/3}(Z)\,, \tag{6.14}$$

where $\langle Z'|\partial_Z^{-1}|Z\rangle = \frac{1}{2}\mathrm{sgn}(Z'-Z)$ and $K = 1+Z^2$. It is zero for $Z_c = \infty$ and 2.377 for $Z_c = \tilde{Z}_c/\sqrt{\lambda}$ in the double limit $\lambda \to \infty$ followed by $\tilde{Z} \to \infty$. See appendix C for details. Thus the charge radius for the nucleon is

$$\sqrt{\langle r^2\rangle_{\mathrm{EM}}} \approx 14.26\left(\frac{1}{2}+I_3\right)M_{\mathrm{KK}}^{-2} \approx 0.784\left(\frac{1}{2}+I_3\right)\mathrm{fm}, \tag{6.15}$$

where we used $M_{\mathrm{KK}} = 950\mathrm{MeV}$ [8]. The experimental values are [15]

$$\sqrt{\langle r^2\rangle_{\mathrm{EM}}^{\mathrm{proton}}} = 0.875\ \mathrm{fm}\,, \qquad \langle r^2\rangle_{\mathrm{EM}}^{\mathrm{neutron}} = -0.1161\ \mathrm{fm}^2\ . \tag{6.16}$$

7. Baryon magnetic moment

The magnetic moments follow from the moments of the electromagnetic current,

$$\begin{aligned}\mu^i &= \frac{1}{2}\epsilon_{ijk}\int d\vec{x}x^j J_{\mathrm{EM}}^k(x) \\ &= \epsilon_{ijk}\int d\vec{y}\sum_n m_{v^n}^2\alpha_{v^n}\psi_{2n-1}(Z_c)\mathcal{Q}_m(\vec{y},Z_c)\int d\vec{x}x^j\Delta_n^{mk}(\vec{y}-\vec{x}) \\ &= \epsilon_{ijk}\int d\vec{y}\sum_n m_{v^n}^2\alpha_{v^n}\psi_{2n-1}(Z_c)\mathcal{Q}_m(\vec{y},Z_c)y^j\frac{-g^{mk}}{m_{v^n}^2} \\ &= -\epsilon_{ijk}\int d\vec{y}y^j\mathcal{Q}^k(\vec{y},Z_c)\,, \end{aligned} \tag{7.1}$$

with

$$\mathcal{Q}_k(\vec{x},Z_c) \equiv \kappa K(Z_c)\mathbb{F}^3{}_{Zk}(\vec{x},Z_c) + \frac{1}{N_c}\kappa K(Z_c)\widehat{\mathbb{F}}_{Zk}(\vec{x},Z_c)\ . \tag{7.2}$$

While the electromagnetic current is meson mediated at the core boundary, its contribution to the magnetic moment to order $\hbar^0$ is core-like owing to the exact sum-rule (4.5) in warped space. By resumming over the tower of infinite vector mesons, the magnetic moment shrunk

[4]The vector-meson coupling to the instanton in the propagator is subleading in $1/\lambda$ and thus dropped.

to the core at strong coupling with g^2 large and N_c large. This remarkable feature is absent in the Skyrme model and its variants since they all truncate the number of mesons.

First consider the iso-scalar contribution to the magnetic moment in (7.2). As we are assessing $\mathcal{Q}_k(x, Z_c)$ at the core boundary, the small Z instanton configuration is sufficient. From (2.15) it follows that

$$\begin{aligned}\frac{1}{N_c}\kappa\widehat{\mathbb{F}}^R_{Zk}(\vec{x}, Z_c) &= \int_{-Z_c}^{Z_c} dZ\left[\frac{1}{32\pi^2}\epsilon_{kNPQ}\Big(\mathrm{tr}\,(\mathbb{F}^R_{N0}\mathbb{F}^R_{PQ})\Big]\\ &= \int_{-Z_c}^{Z_c} dZ\left[\frac{1}{32\pi^2}\epsilon_{kNPQ}\Big(\mathrm{tr}\,(\mathbb{D}_n\mathbb{Z}^R\mathbb{F}_{PQ})\Big]\right.,\end{aligned} \tag{7.3}$$

where we have dropped the U(1) CS contribution as it is subleading as well as surface contributions on R^3_X since we will integrate over R^3_X at the end. The upper R-labels refer to the rigid SO(3) rotation $\mathbb{R}$ and $\mathbb{Z}^R$ is the zero mode from the Gauss constraint (appendix A). In the second line the R-label drops because of tracing. Thus

$$\begin{aligned}\mu^A_{I=0} &= -\epsilon_{Ajk}\int d\vec{y}y^j\frac{2}{N_c}\kappa K(Z_c)\widehat{\mathbb{F}}^R_{Zk}(\vec{x}, Z_c)\\ &= -\frac{\rho^2\omega^A}{4}\frac{Z_c}{\sqrt{Z_c^2+\rho^2}} \to -\frac{\rho^2\omega^A}{4}\\ &= \frac{\langle r^2\rangle'_{I=0}}{6}\frac{J^A}{\mathbb{I}},\end{aligned} \tag{7.4}$$

where we used the BPST solution (2.28) since $Z_c \sim \rho \sim 0$. The contribution is of order $\hbar$ but subleading in $1/\lambda$. The last relation uses that $I^A = -J^A$. This relation for the isoscalar magnetic moment is similar to the one derived in the Skyrme model with the notable difference that only the isoscalar core radius and core moment of inertia are involved.

Now, consider the iso-vector contribution to the magnetic moment. Using (2.12), we have

$$\begin{aligned}\kappa\mathbb{F}^{R,3}_{Zk}(\vec{x}, Z_c) &= \int_{-Z_c}^{Z_c} dZ i\kappa\mathrm{tr}\,\left\{[\mathbb{A}^{R,M}, \mathbb{F}^R_{Mk}]t^3\right\}\\ &\to -\int_{-Z_c}^{Z_c} dZ\frac{\kappa}{4}\mathbb{A}^a_M\mathbb{F}^b_{Mk}\epsilon_{abc}\mathbb{R}_{3c}\,.\end{aligned} \tag{7.5}$$

Much like the iso-scalar, we have only retained the leading contribution to the magnetic moment. As noted earlier, the residual gauge variance of the integrand through Z_C is removed by gauge rotation of the cloud. In terms of the regular BPST solution (2.26) and (2.28) we get

$$\mathbb{A}^a_M\mathbb{F}^b_{Mk}\epsilon_{abc} = \frac{-8\rho^2}{(\xi^2+\rho^2)^3}(x_a\epsilon_{akc} - x_b\epsilon_{kbc})\,, \tag{7.6}$$

so that

$$\begin{aligned}\mu^i_{I=1} &= -\epsilon_{ijk}\int d\vec{y}y^j\kappa K(Z_c)\mathbb{F}^{R,3}{}_{Zk}(x,Z_c)\\ &= -\frac{32\pi}{3}\kappa\rho^2\mathbb{R}_{3i}\int_0^\infty dr\int_{-Z_c}^{Z_c}dZ\frac{r^4}{(\xi^2+\rho^2)^3}\\ &= -4\pi^2\kappa\rho^2\mathbb{R}_{3i}\log_c = -\frac{\mathbb{I}}{2}\mathbb{R}^{3i}\log_c\,, \end{aligned} \tag{7.7}$$

with a logarithmic cutoff sensitivity to the core size,

$$\log_c \equiv \log\left(\frac{Z_c+\sqrt{Z_c^2+\rho^2}}{-Z_c+\sqrt{Z_c^2+\rho^2}}\right)\,. \tag{7.8}$$

The isovector magnetic contribution is of order $\hbar^0$ and similar in structure to the Skyrmion, with the exception that $\mathbb{I}$ is solely driven by the core. The cutoff sensitivity $\log_c$ is absent if we were to use the BPST instanton in the singular gauge. The Cheshire Cat smile survives in the isovector magnetic contribution in the regular gauge (albeit weakly through a logarithm).

Combining the isoscalar and isovector contributions to the magnetic moment yields (singular gauge)

$$\mu^i = \frac{\langle r^2\rangle'_{I=0}}{6}\frac{J^i}{\mathbb{I}} - \frac{\mathbb{I}}{2}\mathbb{R}^{3i}\,, \tag{7.9}$$

which results in the Skyrme-like independent relation [16],

$$\frac{\mu_p-\mu_n}{\mu_p+\mu_n} = \frac{3}{4}\frac{M_\Delta+M_N}{M_\Delta-M_N}\,, \tag{7.10}$$

expected from a soliton. Here $M_{N,\Delta}$ are the nucleon and delta masses split by the inertia $\mathbb{I}$.

8. Conclusions

We have shown how the non-rigid quantization of the D4 instanton in holographic QCD yields baryon electromagnetic form factors that obey the strictures of VMD in agreement with the effective approach discussed in [2]. The holographic baryon at the boundary is composed by a core instanton in the holographic direction at $Z=0$ of size $1/\sqrt{\lambda}$ that is trailed by a cloud of bulk vector mesons and pions of size λ^0 all the way to $Z=\infty$. The core and the cloud interface at Z_C which plays the role of the Cheshire Cat smile (gauge movable). At strong coupling, the baryon size is of order λ^0 thanks to vector meson dominance. The meson-baryon couplings are large and of order $1/\sqrt{\hbar}$ (or lower) and surface-like only owing to the solitonic nature of the instanton.

The electromagnetic form factors, radii and magnetic moments of the ensuing baryons compare favorably with the results obtained in the Skyrme model, as well as data for a conservative value of $M_{\rm KK}=950$ MeV. For instance the electromagnetic charge radii are 0.784 fm (proton) and zero (neutron). They are derived in the triple limit of zero pion

JHEP09(2008)007

mass (chiral limit) and strong coupling λ (large g^2 and large N_c). The magnetic moments are completly driven by the core D4 instanton through a remarkable sum rule of the vector meson cloud. They obey a model independent relation of the type encountered in the Skyrme model, a hall-mark of large N_c and strongly coupled models.

The non-rigid quantization scheme presented here offers a systematic framework for discussing quantum baryons in the context of the semi-classical approximation. It is causal with retardation effects occuring in higher order in $\hbar$. These semiclassical corrections are only part of a slew of other quantum corrections in holography which are in contrast hard to quantify. Also, the small instanton size calls for the use of the full DBI action to characterize the instanton field more faithfully in the holographic core. We note that beyond the $\hbar^0$ contribution discussed here, the issue of Dirac constraints needs to be addressed. This is best addressed in the canonical Hamiltonian formalism whereby Gauss laws are explicitly removed by their constaint equations. The drawback are lack of manifest covariance and operator orderings.

The extension of our analysis to the axial-vector channels is straightforward with minimal changes in our formulae as can be readily seen by inspection. Indeed, the axial vector source differs from the vector one we used by the extra mode function $\psi_0(Z)$ which is odd in Z. Also the pion field Π now contributes. In the axial-vector channel the pion-baryon coupling is expected to be formally of order $\sqrt{\hbar}$ but *time-like* thus effectively of order $1/\sqrt{N_c}$ and not $\sqrt{N_c}$. The Goldberger-Treiman relation in this case follows from the non-rigid quantization of the instanton much like its counterpart for the Skyrmion [12]. Some of these issues and others will be discussed next.

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Note added: upon completion of this work we noted arXiv:0806.3122 [17] where similar issues are addressed with different methods.

A. Gauss law

For the flavor rotated instanton, the rotated form of Gauss law (2.11) reads

$$\mathbb{D}^R_M \mathbb{F}^R_{M0} = \mathcal{O}\left(1/\lambda\right) , \tag{A.1}$$

to leading order as the curvature effects are subleading in $1/\lambda$. All upper R-labels refer to the rigid SO(3) rotation $\mathbb{R}$ with $\mathbb{D}^R_M = \partial_M + \mathbb{A}^{R,A} G^A$. While this is subsumed throughout in the main text, here it is recalled explicitly for the clarity of the argument. The formal solution reads

$$\mathbb{A}^R_0 = \frac{1}{(\mathbb{D}^{R'})^2} \mathbb{D}^R_N \dot{\mathbb{A}}^R_N + \mathbb{Z}^R , \tag{A.2}$$

where the primed inverted operator excludes the zero mode. Following [3], the rotated zero mode solution reads

$$\mathbb{Z}^{R,A} = C\,\mathbb{R}\overline{\mathbb{R}}(g)\omega^A\, f(\xi)\,, \tag{A.3}$$

up to an arbitrary constant C. Here $f(\xi) = \xi^2/(\xi^2+\rho^2)$ and

$$\overline{\mathbb{R}}^{AB}(g) = \mathrm{tr}\left(t^A g^{-1} t^B g\right)\,, \qquad g = \frac{z - i\vec{x}\cdot\vec{\sigma}}{\xi}\,. \tag{A.4}$$

We note that for an unrotated BPST instanton with $\omega^A = 0$, the formal solution (A.2) yields $\mathbb{A}_0 = 0$ as it should. The normalization is $C = 1$ which is fixed by the asymptotic of the $\mathbb{A}^R_0$ field: $\mathbb{A}^R_0(x, Z=\infty) = U^{R\dagger}\partial_0 U^R$. $U = e^{i\tau_a \hat{r}^a(\theta,\phi,\psi)}$ is the identity map and (θ,ϕ,ψ) are the canonical angles on S^3.

In terms of (A.2) the rotated electric field is

$$\mathbb{F}^R_{M0} = \mathbb{D}^R_M \mathbb{A}^R_0 - \dot{\mathbb{A}}^R_M = \mathbb{D}^R_M \mathbb{Z}^R\,. \tag{A.5}$$

The kinetic energy for the rotated instanton is

$$H = \kappa\int d\vec{x}\, dZ\, \mathrm{tr}\,(\mathbb{F}^R_{M0})^2 = \kappa\int d\vec{x}\, dZ\, \mathrm{tr}\,\left(\mathbb{D}_M\mathbb{Z}^R\right)^2 = \frac{1}{2}M_0\rho^2\omega^2\,. \tag{A.6}$$

The upper R-label drops out by tracing. Here $M_0 = 8\pi^2\kappa$ is the instanton mass in units of M_{KK} to leading order in $1/\lambda$.

B. Action

With the gauge field (3.8)–(3.11) the SU(2) YM action reads

$$\begin{aligned} S^{\mathrm{SU}(2)}_{\mathrm{YM}} = -\kappa\int d^4x dZ \mathrm{tr}\,\Big[&\frac{1}{2}K^{-1/3}\mathbb{F}^2_{\mu\nu} + K\mathbb{F}^2_{Z\mu} \\ &+\partial^\mu\left(2K^{-1/3}\mathbb{F}_{\mu\nu}C^\nu\right) + \partial^Z\left(2K\mathbb{F}_{Z\nu}C^\nu\right) + \partial^\mu\left(2K\mathbb{F}_{\mu Z}C^Z\right) \\ &-\left\{\mathbb{D}^\mu\left(2K^{-1/3}\mathbb{F}_{\mu\nu}\right) + \mathbb{D}^Z\left(2K\mathbb{F}_{\mathbb{Z}\nu}\right)\right\}C^\nu - \mathbb{D}^\mu\left(2K\mathbb{F}_{\mu\mathbb{Z}}\right)C^Z \\ &+\frac{1}{2}K^{-1/3}\left\{2\mathbb{F}_{\mu\nu}[C^\mu, C^\nu] + \left(\mathbb{D}_\mu C_\nu - \mathbb{D}_\nu C_\mu - i[C_\mu, C_\nu]\right)^2\right\} \\ &+K\left\{2\mathbb{F}_{Z\mu}[C^Z, C^\mu] + \left(\mathbb{D}_Z C_\mu - \mathbb{D}_\mu C_Z - i[C_Z, C_\mu]\right)^2\right\}\Big]\,, \end{aligned} \tag{B.1}$$

where $\mathbb{D}_\alpha$ is the covariant derivative with the soliton configuration: $\mathbb{D}_\alpha * = \partial_\alpha - i[\mathbb{A}_\alpha, *]$. Similary U(1) YM action is

$$\begin{aligned} S^{U(1)}_{\mathrm{YM}} = -\frac{\kappa}{2}\int d^4x dZ\,\Big[&\frac{1}{2}K^{-1/3}\widehat{\mathbb{F}}^2_{\mu\nu} + K\widehat{\mathbb{F}}^2_{Z\mu} \\ &+\partial^\mu\left(2K^{-1/3}\widehat{\mathbb{F}}_{\mu\nu}\widehat{C}^\nu\right) + \partial^Z\left(2K\widehat{\mathbb{F}}_{Z\nu}\widehat{C}^\nu\right) + \partial^\mu\left(2K\widehat{\mathbb{F}}_{\mu Z}\widehat{C}^Z\right) \end{aligned} \tag{B.2}$$

JHEP09(2008)007

$$
-\left\{\partial^\mu\left(2K^{-1/3}\widehat{\mathbb{F}}_{\mu\nu}\right)+\partial^Z\left(2K\widehat{\mathbb{F}}_{Z\nu}\right)\right\}\widehat{C}^\nu-\partial^\mu\left(2K\widehat{\mathbb{F}}_{\mu Z}\right)\widehat{C}^Z
+\frac{1}{2}K^{-1/3}\left(\partial_\mu\widehat{C}_\nu-\partial_\nu\widehat{C}_\mu\right)^2+K\left(\partial_Z\widehat{C}_\mu-\partial_\mu\widehat{C}_Z\right)^2\Bigg]\,,
$$

and the CS term is

$$
\begin{aligned}
S_{\rm CS} = {} & \frac{N_c}{24\pi^2}\epsilon_{MNPQ}\int d^4x dZ \qquad\qquad (\text{B.3})\\
& \times\Bigg[\frac{3}{8}\left(\widehat{\mathbb{A}}_0+\widehat{C}_0\right)\,\mathrm{tr}\left\{\mathbb{F}_{MN}\mathbb{F}_{PQ}+4\mathbb{F}_{MN}\mathbb{D}_PC_Q+4\mathbb{F}_{MN}C_PC_Q)\right.\\
&\qquad\left.+4(\mathbb{D}_MC_N+C_MC_N)(\mathbb{D}_PC_Q+C_PC_Q)\right\}\\
&\quad-\frac{3}{2}\left(\widehat{\mathbb{A}}_M+\widehat{C}_M\right)\,\mathrm{tr}\left\{\partial_0(\mathbb{A}_N+C_M)(\mathbb{F}_{PQ}+2\mathbb{D}_PC_Q+2C_PC_Q)\right\}\\
&\quad+\frac{3}{4}\left(\widehat{\mathbb{F}}_{MN}+2\partial_M\widehat{C}_N\right)\,\mathrm{tr}\left\{(\mathbb{A}_0+C_0)(\mathbb{F}_{PQ}+2\mathbb{D}_PC_Q+2C_PC_Q)\right\}\\
&\quad+\frac{1}{16}\left(\widehat{\mathbb{A}}_0+\widehat{C}_0\right)\left\{\widehat{\mathbb{F}}_{MN}\widehat{\mathbb{F}}_{PQ}+4\widehat{\mathbb{F}}_{MN}\partial_P\widehat{C}_Q+4(\partial_M\widehat{C}_N)(\partial_P\widehat{C}_Q)\right\}\\
&\quad-\frac{1}{4}\left(\widehat{A}_M+\widehat{C}_M\right)\left\{\widehat{\mathbb{F}}_{0N}\widehat{\mathbb{F}}_{PQ}+2\widehat{F}_{0N}\partial_P\widehat{C}_Q+\widehat{F}_{PQ}(\partial_0\widehat{C}_N-\partial_N\widehat{C}_0)\right.\\
&\qquad\left.+2(\partial_0\widehat{C}_N-\partial_N\widehat{C}_0)\partial_P\widehat{C}_Q\right\}\\
&\quad+\frac{3}{2}\partial_N\left[\left(\widehat{\mathbb{A}}_M+\widehat{C}_M\right)\mathrm{tr}\left\{(\mathbb{A}_0+C_0)(\mathbb{F}_{PQ}+2\mathbb{D}_PC_Q+2C_PC_Q)\right\}\right]\Bigg]\\
&+\frac{N_c}{48\pi^2}\int\mathrm{d}\left[\left(\widehat{\mathbb{A}}+\widehat{C}\right)\,\mathrm{tr}\left\{2\mathrm{d}(\mathbb{A}+C)(\mathbb{A}+C)-\frac{3i}{2}(\mathbb{A}+C)^3\right\}\right]\,.
\end{aligned}
$$

C. Integral

In this appendix we work out the integral in (6.14):

$$
G(Z_c)\equiv-\int dZ\int dZ'\langle Z_c|\partial_Z^{-1}|Z'\rangle K^{-1}(Z')\langle Z'|\partial_Z^{-1}|Z\rangle K^{-1/3}(Z)\,. \qquad (\text{C.1})
$$

We start with the Green function

$$
\langle Z'|\partial_Z^{-1}|Z\rangle=\frac{1}{2}\mathrm{sgn}(Z'-Z)\,. \qquad (\text{C.2})
$$

Since there are two sgn functions in (C.1) we divide the integral region into six pieces reflecting all possible sign difference:

$$
\begin{aligned}
G(Z_c) = -\frac{1}{4}\Bigg[&\int_{-\infty}^{Z_c}dZ'\int_{-\infty}^{Z'}dZ-\int_{-\infty}^{Z_c}dZ'\int_{Z'}^{Z_c}dZ \qquad (\text{C.3})\\
&-\int_{Z_c}^{\infty}dZ'\int_{Z_c}^{Z'}dZ+\int_{Z_c}^{\infty}dZ'\int_{Z'}^{\infty}dZ\\
&-\int_{Z_c}^{\infty}dZ'\int_{-\infty}^{Z_c}dZ-\int_{-\infty}^{Z_c}dZ'\int_{Z_c}^{\infty}dZ\Bigg](K^{-1}(Z')K^{-1/3}(Z))\,.
\end{aligned}
$$

JHEP09(2008)007

In two extreme(symmetric) case the expression becomes simple. For $Z_c = \infty$

$$-\frac{1}{4}\left[\int_{-\infty}^{\infty} dZ' \int_{-\infty}^{Z'} dZ - \int_{-\infty}^{\infty} dZ' \int_{Z'}^{\infty} dZ\right](K^{-1}(Z')K^{-1/3}(Z)) = 0\,, \tag{C.4}$$

and for $Z_c = 0$

$$-\frac{1}{2}\left[\int_{-\infty}^{0} dZ' \int_{-\infty}^{Z'} dZ - \int_{-\infty}^{0} dZ' \int_{Z'}^{0} dZ - \int_{0}^{\infty} dZ' \int_{-\infty}^{0} dZ\right](K^{-1}(Z')K^{-1/3}(Z)) \tag{C.5}$$

$$= \int_0^{\infty} dZ' K^{-1}(Z') \int_0^{Z'} dZ K^{-1/3}(Z) \sim 2.377\ .$$

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JHEP09(2008)007